Optimal Selling Policy for Supply Chain System with Returns
Handling, Clearance and Secondary Sales Markets and Profit Sharing

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Abstract In this paper, we present an optimal selling policy for a supply chain system with return handling, clearance sale and profit sharing. In our model, a supply chain system consists of a manufacturer and a retailer. A single type of product is sold in four consecutive periods: the normal sale period, the clearance sale period, the secondary sale period and the disposal sale period. As a selling policy, we focus on product order quantity and discount rate for regular product price. We analyze two types of supply chain: (Type I) the decentralized supply chain, (Type II) the centralized supply chain. In Type I, the optimal decision is made to maximize the retailer’s expected profit. In Type II, the optimal decision is made to maximize the whole system’s expected profit. We also discuss supply chain coordination for the manufacturer-retailer partnership based on profit sharing. The profit sharing is permitted only in Type II as supply chain coordination contract between the manufacturer and the retailer. In numerical examples, the effect of supply chain coordination are shown, by comparing the expected profits of each player and the whole system in Type I with those in Type II with profit sharing.

Keywords: E-commerce, Return handling, Clearance and secondary sales, Profit sharing, Decentralized supply chain, Centralized supply chain, News-vender type model

1. INTRODUCTION
For many years, return handling approaches for unsold products or profit sharing approaches between a manufacturer and a retailer have been discussed. For examples of return handling, resale of returns, buy-back, sale of disposal sale market, secondary sale market or markdown sale market. (Vlachos&Dekker(2003), Choi&Yan, Lee(2007), Niimi et al.(2009)). However, the combination of above all return handleings and profit sharing has not been sufficiently discussed in past literatures.

In this paper, we present an optimal selling policy for a supply chain with return handling, clearance sale and profit sharing. A single type of product is sold in four consecutive periods: the normal sale period, the clearance sale period, the secondary sale period and the subsequent leftovers disposal sale period. We analyze two types of supply chain: (Type I) the decentralized supply chain where a retailer is the Stackelberg leader and a manufacturer is the Stackelberg follower, (Type II) the centralized supply chain where a manufacturer and a retailer are integrated as one company. In the decentralized supply chain, the optimal decision is made to maximize the retailer’s expected profit. In the centralized supply chain, the optimal decisions are made to maximize the whole system’s expected profit which is the sum of each player’s expected profit. We also discuss a supply chain coordination contract to obtain the manufacturer-retailer partnership based on profit sharing. The profit sharing contract is permitted only in the centralized system between the manufacturer and the retailer. In numerical examples, the effect of supply chain coordination is shown, by comparing the expected profits of each player and the whole system in the decentralized system with those in the centralized system with profit sharing.

2. NOTATION AND MODEL ASSUMPTIONS

\[ Q_i(i = D, C) \]: product order quantity of retailer in either the decentralized supply chain \((i = D)\) or the centralized supply chain \((i = C)\).
\[ x_k(k = 1, 2, 3) \]: customer’s demand of product which either a retailer faces in either normal sale market \((k = 1)\) and clearance sale market \((k = 2)\) or a manufacturer faces.

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2. MODEL DESCRIPTIONS

2.1 Model Assumptions
(1) An intended product is a single type and a single period product. A single decision for the order quantity is to be made before the beginning of the selling period in the normal sale period so as to maximize the Stackelberg leader’s expected profit.
(2) A retailer faces an uncertain demand \( x_1 \) of the product in the normal sale market. Also, a retailer faces an uncertain demand \( x_2 \) of the product in the clearance sale market. A manufacturer faces an uncertain demand \( x_3 \) of the product in the secondary sale market. \( x_1 \), \( x_2 \) and \( x_3 \) follow the independent demand distributions with the probability density function \( f_i(\cdot) \) and the cumulative distribution function \( F_i(\cdot) \) for the demand \( x_i \) \((k=1,2,3)\).

2.2 Model Operations
Figure 1 shows system operations of our supply chain with return handling and clearance sale.
(1) A manufacturer produces a single type of product with \( c \) per unit. The manufacturer sells the products to a retailer with \( w \) per unit.
(2) The retailer sells the products to a customer in the normal sale market with \( u \) per unit. When either the excess inventory or the shortage of the inventory occurs, the retailer incurs the inventory holding cost \( h_{1i} \) per unit per unit time or the shortage penalty cost \( G \) per unit.
(3) During the normal sale period, some products may be returned to the retailer from the customer at a fixed return rate \( r \). It is allowed that the return for the same product is acceptable once only to the retailer within the normal sale period. The retailer inspects all the returned products. The returned products are or can be in the as-good-as-new condition after the recovery by the retailer if necessary. After inspecting and recovering the returned products, the retailer can resell all the returned products in the normal sale market.
(4) After the end of the normal sale period, the retailer sells the unsold products at a discount sale price \((1-d)u\) per unit in the clearance sale market.
(5) After the end of the selling period in the clearance sale market, the manufacturer buys back the unsold products from the retailer with \( b \) per unit.
(6) The manufacturer reworks the unsold products with \( m \) per unit. After reworking, the manufacturer sells the unsold products to the secondary market with \( u_{2bd} \) per unit. When the excess inventory occurs, the manufacturer incurs the inventory holding cost.
(7) After the secondary sale period, the manufacturer sells the unsold products to the disposal sale market with \( v \) per unit of product.
3. MODEL FORMULATIONS

3.1 The Manufacturer’s Expected Profit

Based on Figure 1 and model assumptions in 2.1 and model operations in 2.2, the profit of the manufacturer consists of the production cost of the product, the sale of the product to the retailer, the buy-back cost of the unsold products to the retailer after the end of the clearance sale period, the rework cost of the unsold products to sell in the secondary sale market, the sale of the unsold product to the secondary sale market, the disposal sale of the unsold products in the disposal sale market. In consideration of the relation between the product order quantity $Q_o$ from the retailer, the demand of the product $x_1$ in the normal sale market, the demand of the product $x_2$ in the clearance sale market and the demand of the product $x_3$ in the secondary sale market, the expected profit of the manu-facturer is formulated as

$$E[\pi_m(Q_o)] = (w-c)Q_o + \{u_{SN} - b - m - h_m\}
\times \int_0^{\Theta} \left[ \int_0^{\Theta - z} \left( Q_o - x_1 - x_2 \right) f_2(x_2) dx_2 \right] f_1(x_1) dx_1
+ \{v - (u_{SN} - m - h_m)\} \int_0^{\Theta} \int_0^{\Theta - z} \int_0^{\Theta - z - z} \left( Q_o - x_1 - x_2 \right) f_2(x_2) dx_2 f_1(x_1) dx_1.
\tag{1}$$

3.2 The Retailer’s Expected Profit

The profit of the retailer consists of the purchase cost of the product from the manufacturer, the sale of the product in the normal sale market, the return handling cost of the returned products, the inventory holding cost of the unsold product after the end of the normal sale period, the shortage penalty cost of the product in the normal sale market, the sale of the product in the clearance sale market, the maintenance cost of the unsold products for clearance sale, the inventory holding cost of the unsold products after the end of the clearance sale period, the buy-back revenue of the unsold products from the manufacturer. In consideration of the relation between the product order quantity $Q_o$ of the retailer and the demands $x_1$ and $x_2$ of the product in the normal sale market and the clearance sale market, the retailer’s expected profit is formulated as

$$E[\pi_e(Q_o)]
= ((1-r)u - (s + C_s + qC_s) r + ru) \int_0^{\Theta} x_1 f_1(x_1) dx_1
+ (-h_1 + (1-d)u - c_s) \int_0^{\Theta} f_1(x_1) dx_1
+ (b - (-h_1 + (1-d)u - c_s)) h_2
\times \int_0^{\Theta} \int_0^{\Theta - z} \left( Q_o - x_1 - x_2 \right) f_2(x_2) dx_2 f_1(x_1) dx_1
+ (1-r)u - (s + C_s + qC_s) r + ru \int_0^{\Theta} f_1(x_1) dx_1
- G \int_0^{\Theta} (x_1 - Q_o) f_1(x_1) dx_1 - wQ_o.
\tag{2}$$

3.3 The Whole System’s Expected Profit

For a given quantity $Q_o$, the whole system’s expected profit is obtained as the sum of the expected profits of the manufacturer and retailer in (1) and (2).

$$E[\pi_s(Q_o)] = E[\pi_m(Q_o)] + E[\pi_e(Q_o)].
\tag{3}$$

4. OPTIMAL DECISION

4.1 Decision of Decentralized Supply Chain

The optimal order quantity $Q_o$ of the decentralized supply chain is determined so as to maximize the...
Stackelberg leader (retailer)”s the expected profit in equation (2). Therefore, the optimal product order quantity $Q_o$ can be obtained as the approximate-solution by searching in the range satisfying both the conditions of

$$dE[\pi_e(Q_o)]/dQ_o = 0. \quad (4)$$

$$d^2E[\pi_e(Q_o)]/dQ_o^2 < 0.$$ Substituting the optimal product order quantity $Q_o$ determined in equation (4) into equations (2), (1) and (3), the retailer’s maximal expected profit, the expected profits of the manufacturer and the whole system in the decentralized supply chain can be obtained.

4.2 Decision of Centralized Supply Chain

In the centralized supply chain, the optimal decision is made under a situation where the manufacturer and the retailer are integrated as one company. The optimal product order quantity $Q_c$ of the product is determined to maximize the expected profit of the whole system. In consideration of the relation between the product order quantity $Q_c$ of the retailer and the demands, $x_1$, $x_2$ and $x_3$, of the product in normal sale market, the clearance sale market and the secondary sale market, the whole system’s profit $\pi_c(Q_c)$ in the centralized supply chain is obtained as

$$E[\pi_c(Q_c)] = E[\pi_{sd}(Q_c)] + E[\pi_{sh}(Q_c)]$$

$$= \left[ (1-r)u_i-(s+C_r+qC_r) + r u_i \right] \int_0^Q x_i f_i(x_i) dx_i$$

$$+ \left[ (-h_i + (1-d)u_i - c_d) \right] \int_0^Q (Q_c - x_i) f_i(x_i) dx_i$$

$$+ \left[ (-h_i + (1-d)u_i - c_d) - h_z + (u_i - m_z - h_z) \right] \int_0^Q \left[ \int_0^{x_i} (Q_c - x_i - f_j(x_j) dx_j \right] f_i(x_i) dx_i$$

$$= \left[ (1-r)u_i-(s+C_r+qC_r) + r u_i \right] \int_0^Q x_i f_i(x_i) dx_i$$

$$+ \left[ (-h_i + (1-d)u_i - c_d) \right] \int_0^Q (Q_c - x_i) f_i(x_i) dx_i$$

$$-G \int_0^Q (x_i - Q_c) f_i(x_i) dx_i - m_i Q_c$$

$$+ \left[ v - (u_i - m_z - h_z) \right] \int_0^Q \int_0^{Q_c - x_i} (Q_c - x_i - x_j) f_i(x_i) dx_i$$

$$\times f_j(x_j) dx_j f_j(x_j) dx_j.$$ (5)

The optimal product order quantity $Q_c$ of the centralized supply chain is determined so as to maximize the whole system’s the expected profit in equation (5). Therefore, the optimal product order quantity $Q_c$ can be obtained as the approximate-solution by searching in the range satisfying both the conditions of

$$dE[\pi_c(Q_c)]/dQ_c = 0. \quad (6)$$

$$d^2E[\pi_c(Q_c)]/dQ_c^2 < 0.$$ Substituting the optimal product order quantity $Q_c$ determined in equation (6) into equations (5), (1) and (2), the whole system’s maximal expected profit, the expected profits of the manufacturer and the retailer in the centralized supply chain can be obtained.

5. PROFIT SHARING

We use a simple profit sharing approach as to each player’s investment ratio, based on Chauhan & Proth’s model (2005). If there is increment of the whole system’s expected profit in the centralized supply chain, the amount of profit sharing is given to all players under the centralized supply chain. First, the investment ratio is calculated as the proportion of each player’s expected profit to each player’s expected cost under the centralized supply chain. Next, each player’s investment ratio is normalized so that the sum of all players’ investment ratios is 1. The amount of profit sharing as to each player’s investment ratio is added to each player’s expected profit in the decentralized supply chain where the Stackelberg leader’s optimal decision is adopted.

6. NUMERICAL EXAMPLES

In this section, we discuss the effect of clearance sale on the optimal order quantity and the expected profits of the manufacturer, the retailer and the whole system. Concretely, we compare the results in the decentralized supply chain with return handling without clearance (base model) with those with both return handling and clearance sale (our model). Similarly, in the centralized supply chain, we compare the results in base model with those in our model. The optimal decisions without clearance sale under both the decentralized and centralized supply chain can be made by removing all terms related to clearance sale in equation (1) and (2). We discuss the effect of profit sharing on expected profits of the manufacturer and the retailer in the decentralized supply chain. We compare the optimal order quantity and the expected profits of both players in the decentralized supply chain with those in the centralized supply chain with profit sharing.

System parameters used in numerical examples are set as follows: The demand $x_1$ of the product in the normal sale market follows the normal distribution with population mean $\mu = 1000$ and population standard deviation $\sigma_1 = 300$. The demand $x_2$ of the product in the clearance sale market follows the normal distribution with population mean $\mu_2 = d \times \mu_1$, where $\mu_2 = 0.3 \times \mu_1$, and population standard deviation $\sigma_2 = d \times \sigma_1$, where $\sigma_2 = 0.3 \times \sigma_1$, depending on the value of discount ratio.
Table 1: Comparison results of base model with return handling without clearance sale and our model with both return handling and clearance sale under the decentralized supply chain.

<table>
<thead>
<tr>
<th>Discount ratio ( d )</th>
<th>Optimal order quantity</th>
<th>Maximal expected profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manufacturer</td>
<td>Retailer</td>
</tr>
<tr>
<td>----</td>
<td>1153</td>
<td>1958.09</td>
</tr>
<tr>
<td>0.1</td>
<td>1105</td>
<td>1916.73</td>
</tr>
<tr>
<td>0.2</td>
<td>1190</td>
<td>2067.33</td>
</tr>
<tr>
<td>0.3</td>
<td>1180</td>
<td>2076.53</td>
</tr>
<tr>
<td>0.4</td>
<td>1182</td>
<td>2108.57</td>
</tr>
<tr>
<td>0.5</td>
<td>1177</td>
<td>2129.38</td>
</tr>
<tr>
<td>0.9</td>
<td>1099</td>
<td>2101.79</td>
</tr>
</tbody>
</table>

Table 2: Comparison results of base model with return handling without clearance sale and our model with both return handling and clearance sale under the centralized supply chain.

<table>
<thead>
<tr>
<th>Discount ratio ( d )</th>
<th>Optimal order quantity</th>
<th>Maximal expected profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manufacturer</td>
<td>Retailer</td>
</tr>
<tr>
<td>----</td>
<td>1197</td>
<td>1996.26</td>
</tr>
<tr>
<td>0.1</td>
<td>1130</td>
<td>1935.43</td>
</tr>
<tr>
<td>0.2</td>
<td>1242</td>
<td>2116.52</td>
</tr>
<tr>
<td>0.3</td>
<td>1229</td>
<td>2122.98</td>
</tr>
<tr>
<td>0.4</td>
<td>1236</td>
<td>2163.34</td>
</tr>
<tr>
<td>0.5</td>
<td>1235</td>
<td>2192.15</td>
</tr>
<tr>
<td>0.9</td>
<td>1169</td>
<td>2203.42</td>
</tr>
</tbody>
</table>

Table 3: Comparisons of the optimal order quantity and the expected profits in the centralized supply chain without profit sharing and those in the centralized supply chain with profit sharing.

<table>
<thead>
<tr>
<th>Discount ratio ( d )</th>
<th>Optimal order quantity</th>
<th>Maximal expected profit</th>
<th>Profit sharing for centralized supply chain</th>
<th>Non-profit sharing for centralized supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Manufacturer</td>
<td>Retailer</td>
<td>Manufacturer</td>
</tr>
<tr>
<td>0.2</td>
<td>1242</td>
<td>7055.43</td>
<td>2076.77</td>
<td>4978.66</td>
</tr>
<tr>
<td>0.4</td>
<td>1236</td>
<td>7085.42</td>
<td>2119.56</td>
<td>4965.86</td>
</tr>
</tbody>
</table>

\( d \) of sale price \( u \). The demand \( x_i \) of the product in the secondary sale market follows the normal distribution with population mean \( \mu_i = 0.1 \times \mu_1 \) and population standard deviation \( \sigma_i = 0.1 \times \sigma_1 \). As the system parameters of a manufacturer: \( c=5.0, \quad w=7.0, \quad b=3.0, \quad a_{2N} = 3, \quad m=1.0, \quad h_0=1.0, \quad v=1.5 \). As the system parameters of a retailer: \( u=14.0, \quad r=0.2, \quad s=0.5, \quad C_t=1.0, \quad q=0.8, \quad C_e=2.0, \quad G=5.0, \quad h_1=h_2=1.0, \quad C_r=1.5, \quad 0.0 \leq d \leq 1.0 \).

Table 1 shows comparison results of base model and our model under the decentralized supply chain. Table 2 shows comparison results of base model and our model under the centralized supply chain without profit sharing. For discount ratio \( d \), ‘----’ indicates \( d=0.0 \) (base model). The range of \( 0.1 \leq d \leq 1.0 \) indicates our model. From Tables 1 and 2, the optimal order quantities of our model with clearance sale under both the decentralized supply chain and the centralized supply chain are larger than those of base model without clearance sale. From Tables 1 and 2, the followings can be seen: all the expected profits of the manufacturer, the retailer and the whole system of our model with clearance sale under both the decentralized supply chain and the centralized supply chain are higher than those of base model. Under the decentralized supply chain, when \( 0.2 \leq d < 0.3 \), the retailer’s maximal expected profit is obtained. Under the centralized supply chain, when \( 0.4 \leq d < 0.5 \), the whole system’s maximal expected profit is obtained. We discuss the causes as follows:

(1) Behavior of the optimal product order quantity for the value of discount ratio \( d \).

From Tables 1 and 2, the followings can be seen: in the centralized supply chain, when \( d>0.4 \), the optimal order quantity reduces gradually. In the decent-
ralized supply chain, when \( d > 0.2 \), the optimal product order quantity reduces gradually. This is the reason the profit obtained from the sales of the products reduces as the discount ratio \( d \) increases. Also, when \( 0.2 \leq d \leq 0.6 \) under the decentralized supply chain, the expected profit of the manufacturer increases. When \( 0.2 \leq d \leq 0.7 \) under the centralized supply chain, so does it. This is the reason the buy-back quantity of the unsold products from the retailer reduces in this range of \( d \). However, when the \( d \) is larger than 0.6 or 0.7, the expected profit of the manufacturer reduces gradually. This is the reason the profit obtained from the sales of the unsold products reduces in this range of \( d \).

(2) Behaviors of clearance sales, secondary sales and disposal sales for the discount ratio \( d \)

From Tables 1 and 2, the followings can be seen: under both the decentralized supply chain and the centralized supply chain, when \( 0.0 \leq d \leq 0.2 \) and \( 0.8 \leq d \leq 0.9 \), the expected profit of the clearance sale market is deficit. This is the reason the expected costs consisting of the inventory holding cost and the maintenance cost for the unsold product are higher than the expected sales of the unsold product in this range of \( d \). The larger \( d \) is, the less the expected profits of the secondary sale market and the disposal sale market are.

(3) Effect of profit sharing on each player’s profit

In both base model and our model, the expected profits of the manufacturer who is the Stackelberg follower and the whole system under the centralized supply chain are higher than those in the decentralized supply chain. However, the expected profit of the retailer in the centralized supply chain is lower than that of the retailer who is the Stackelberg leader in the decentralized supply chain. Table 3 shows the results without/with profit sharing in the centralized supply chain for base model and our model. In Tables 2 and 3, we compare the results of all the expected profits in the centralized supply chain without profit sharing with those with profit sharing. From the results, after the profit sharing, the expected profits of the retailer and the manufacturer under the centralized supply chain are higher than those in the decentralized supply chain.

7. CONCLUSION

In this paper, we presented an optimal selling policy for a supply chain with return handling, clearance sale and profit sharing. The optimal discount ratio and profit sharing can bring the more profit to all players and the whole system under the centralized supply chain. As future research, it will be necessary to discuss another profit sharing approach so as to adjust contract parameters for supply chain coordination.

ACKNOWLEDGMENT

This research has been supported by the Grant-in-Aid for Scientific Research (No. 20510138) from the Japan Society for the Promotion of Science.

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